This paper provides the first estimate of an error variance-covariance matrix for altimeter measurements of global mean sea level rise. The authors then derive a 90% confidence interval of GMSL on a 10-day basis and estimate the trend and acceleration of GMSL over 5 year or longer intervals. Overall the paper is easy to understand and could potentially provide a useful quantification of uncertainty. However, my primary concern is with the treatment of GIA uncertainty and the authors must address this.

We thank reviewer 1 for this positive review. In the revised manuscript and the detailed response below we now address reviewer 1's concern about the treatment of the GRD correction associated to present day mass loss. We thank reviewer 1 for pointing us to this flaw in the manuscript.

The authors note that they use the Spada 2017 estimate of 0.05 mm/year for GIA uncertainty. This uncertainty estimate is for the GIA component due to the ongoing changes in the Earth’s crust since the last glacial maximum (LGM) but does not include modern day melt contributions to GIA. As the authors are aware, the LGM-GIA response is typically accounted for in altimeter-based estimates of GMSL by adding 0.3 mm/yr to the altimeter-derived estimate of GMSL. However, this estimate does not account for deformations of the ocean bottom due to modern melt, which can introduce biases in both the mean trend and acceleration term. See, for example, Frederikse et al. 2017 and Lickley et al. 2018. This correction need not be included if the authors wish to use altimeter measurements to estimate changes in sea surface height instead of sea level. However, the authors explicitly reference estimates of changes in sea level (lines 117-120) where they compare altimeter estimates of GMSL to changes in ocean volume as measured by tide gauges, or the sum of the contributions to changes in ocean volume. To be consistent, I believe this additional source of GIA uncertainty should be accounted for. Alternatively, they could remove the GIA estimate altogether and state upfront that this is an estimate of the uncertainty in sea surface height and cannot be compared to volumetric changes in sea level.

Reviewer 1 is right, we need to include the Frederikse et al. (2017) and Lickley et al. (2018) elastic correction in our study because we compare altimeter estimates of GMSL rise to changes in ocean volume as measured by tide gauges. We now correct our estimate of the GMSL rise by +0.10 mm/yr (in the text and in figures 1, 4 and 9) as recommended by Frederikse et al. (2017). The uncertainty in this correction arises mainly from uncertainty associated to the procedure to solve the sea level equation, uncertainty in the choice of the Love numbers, uncertainty generated by the truncation degree of the spherical harmonics and the uncertainty in the mass redistribution. Because the elastic response of the Earth and its main parameters...
(i.e. the sea level equation, the Love numbers, the spherical harmonic development) are reasonably well defined (Mitrovica et al., 2011), the uncertainty in this correction is largely dominated by uncertainties in the mass redistribution (Frederikse et al. 2017). The uncertainty on the mass redistribution is about ±10% on the current ice mass loss (e.g. Blazquez et al. 2018, The WCRP sea level budget group 2018). Since the elastic response of the solid Earth is linear, the uncertainty in the ocean bottom motion associated to the uncertainty in the mass redistribution should also amount ±10% of the total correction. It yields an uncertainty of ±0.01 mm/yr on the elastic correction. This uncertainty is very small. It is an order of magnitude smaller than the uncertainty considered in this study (see Table 1). So we neglect this source of uncertainty in our study. We now write a paragraph on line 335 to explain this.

Specific Comments: There are a number of grammatical errors and issues with vocabulary choice throughout. Please check!

Here are a few examples:

Line 41: add an s to “altimeter” Replace “confidence enve-lope” with “confidence interval”

throughout Line 86: replace “the GMSL” with “GMSL”.

Line 96: Add “us” after “enables” and remove the “s” on “metrics”

Other issues: Line 307, should be ‘∼’ not ‘=’ Please label axes on Figure 5 and 9.

Anonymous Referee #2

Received and published: 12 June 2019

General comments

In this paper the uncertainty in the satellite estimate of Global Mean Sea Level changes, particularly referring to the trend and the acceleration has been evaluated. I have read it with attention, finding that its quality is quite good, in my opinion. The English form is generally good but at some sections it needs to be further improved. Moreover, the research group appears to be qualified in the field of satellite oceanography. Nonetheless this, a moderate revision is still necessary for a further improvement of the paper’s quality (see specific comments).
We thank reviewer 2 for this positive review. In the revised manuscript and the detailed response below we now address the miswording. We thank reviewer 2 for the detailed reading of our manuscript and for the rewording suggestions.

The topic of Global Mean Sea Level and its relationships with climate changes has been deeply studied in marine geophysics and satellite oceanography (Ablain et al., 2015; 2017; Abraham et al., 2013; Allan et al., 2014; Aucan et al., 2017; Baki Iz et al., 2018; Beckley et al., 2010; 2017; Boening et al., 2012; Cazenave et al., 2014; Chambers et al., 2010; Chen et al., 2017a; 2017b; Church and White, 2006; 2011; Church et al., 2013; Clark and Primus, 1987; Conrad and Hager, 1997; Curry, 2018; Dahlen, 1976; Dangendorf et al., 2017; Davis and Mitrovica, 1996; Desai et al., 2015; Desbruyeres et al., 2016; Dieng et al., 2017; Esselborn et al., 2018; Farrell and Clark, 1976; Fasullo et al., 2013; 2016; Frederikse et al., 2017a; 2017b; 2018; Gardner et al., 2013; Gornitz et al., 2019; Gregory et al., 2013; Haigh et al., 2014; Hamlington and Thompson, 2015; Hamlington et al., 2013; 2016; 2017; Handoko and Hariyadi, 2018; Hay et al., 2015; Herring et al., 2019; Kay et al., 2014; Kendall et al., 2005; Kidwell et al., 2017; Lickley et al., 2018; Melachroinos et al., 2013; Merrifield et al., 2009; Milne and Mitrovica, 1996; Mitchum, 2000; Mitrovica and Milne, 2003; Mitrovica et al., 2001; Nerem and Fasullo, 2018; Nerem et al., 1999, 2010; Prandi et al., 2009; Ray et al., 2013; Shepherd et al., 2018; Slangen et al., 2016; 2017; Swart et al., 2015; Spada, 2017; Spada and Galassi, 2016; Tamisiea, 2011; Thompson et al., 2016; Trenberth et al., 2016; Vaughan et al., 2013; Wahr et al., 2015; Wang et al., 2017; Watkins et al., 2015; Watson et al., 2015; Wiese et al., 2016; Wouters et al., 2013).

Due to the exceptional abundance of recent scientific literature addressing this research topic, I suggest perhaps to the authors to expand the discussion of their results, taking into account some of the scientific papers listed in the attached references, which have not considered in detail. This could be a general issue to be addressed in the revision of the manuscript.

We thank reviewer 2 for suggesting all these publications. This abundant literature address many different scientific questions such as

1) general climate variability (Herring et al., 2019; Kay et al., 2014; Swart et al., 2015)
2) coastal sea level (Kidwell et al., 2017, Prandi et al., 2009)
3) the closure of the sea level budget (Boening et al., 2012; Cazenave et al., 2014, Chen et al., 2017a, Dieng et al., 2017, Fasullo et al., 2013, Watson et al., 2015)
4) the 20th century sea level changes (Aucan et al., 2017, Church and White, 2006; 2011, Dangendorf et al., 2017 Frederikse et al., 2017a, 2018 Gregory et al., 2013, Hamlington and Thompson, 2015, Hay et al., 2015, Ray et al., 2013, Slangen et al. 2017, Thompson et al., 2016)
5) the contributions to sea level change (Abraham et al., 2013, Chambers et al., 2010, Cheng et al. 2017b, Gardner et al., 2013, Desbruyeres et al., 2016, Shepherd et al., 2018, Wiese et al., 2016, Conrad and Hager, 1997; Hamlington et al., 2013; 2016; 2017; Nerem et al., 1999, Wang et al., 2017; Watkins et al., 2015, Wouters et al., 2013)
7) the acceleration in sea level during the altimetry period (Fasullo et al. 2016; Haigh et al., 2014, Nerem and Fasullo, 2018; Nerem et al. 2018)
We want to highlight here that this paper focuses on the uncertainties in sea level estimates from satellite altimetry. As such only a few of these publications are actually relevant for our purpose. Those are the ones related to the scientific questions number 7 and 8. We now consider these publications and include them in our manuscript (except for Haigh et al. 2014 and Nerem and Fasullo 2018 which address the question of the acceleration in the sea level response to GHG emissions while we address in our paper the question of sea level changes in response to any forcing and to internal variability. As such these two publications are not relevant to our paper). We thank reviewer 2 for pointing these missing references.

Moreover, the relationships among the sea level changes and the subsidence of the basin, both to a regional and to a local scale have not been analyzed. I suggest perhaps to add in the discussion a short paragraph (half one page) clarifying which are the relationships existing between the oceanographic aspects and the geological processes controlling the sea level fluctuations. This discussion will represent a main added value further improving the quality of the paper. In particular, I think that the relationships between the water column and the height of the sea bottom, as controlled by subsidence, both isostatic and tectonic, need to be clarified.

This is done now on line 365

I suggest to the authors to carefully avoid the English grammar repetition and to avoid to be redundant, as it happens in some sections of this manuscript.

We have the English grammar mistakes in the revised revision. Please see the specific comments below

Specific comments
I suggest to eliminate the quotations of references in the abstract of the paper. Usually, the abstract does not include any quotation.
I suggest to put the quotation of references in the paper in a chronological order, not alphabetical one, if not strictly required from the journal.

This is not possible as the journal requires an alphabetical order.

The discussed needs to be expanded taking into account recent literature and geological aspects, as mentioned in the general comments.

The conclusions need to be consequently expanded.

Please see our answer to the general comments above.

The captions to figures need to be carefully revised and corrected.

Done as suggested by reviewer 2 in his specific comments. Please see our answer to the specific comments.

Abstract
Row 17
.....anthropogenic activity, or estimating the Earth’s energy imbalance. Previous authors have estimated the uncertainty.... and have shown that it amounts to.....

In this study, we extend our previous results providing a comprehensive description of the uncertainties in the satellite GMS record. We analyzed ..... and estimated...ten days.

Three types of errors have been modeled (drifts, biases, noises) and combined together to derive a realistic estimate of the GMSL error variance-covariance matrix.

We derived a 90% confidence envelop of the GMSL record on a 10-day basis from the error variance-covariance matrix.

Then we used a least squared approach ....

Over 1993-2017 we have found a GMSL trend...
I suggest to eliminate this sentence.

Moreover, in the abstract there is the repetition of the term “estimating”. Try to avoid it.

1. Introduction

The sea level change is a key indicator of global climate change, which integrates changes in several components of the climatic system as a response to climatic variability, both anthropogenic and natural.

Six research groups (AVISO/CNES, SL_cci/ESA, University of Colorado, CSIRO, NASA/GSFC, NOOA) have processed the sea level raw data provided by satellite altimetry to estimate the GSML series on a 10-day basis (Figure 1)

There is a repetition of the terms difference and different. Try to re-write this paragraph avoiding the repetition.

different interpolation methods applied by several groups (Masters et al., 2012; Henry et al., 2014).

This spread is smaller than the real uncertainty in the sea level trend, because all the research groups have used similar methods and corrections to process the raw data and thus several sources of systematic uncertainty are not accounted for in the spread.

In a previous study Ablain et al. (2009) have proposed a realistic estimate of the uncertainty in the GMSL trend over 1993-2008, using an approach based on the error budget. They have identified the radiometer wet tropospheric correction as one of the main sources of error. They have also proposed the orbital determination……
In the framework of the ESA Sea Level Climate Change Initiative (SL_cci), significant improvements have been obtained estimating the sea level from space (Ablain et al., 2015; Quartly et al., 2017; Legeais et al., 2018) to get closer to the GCOS requirements.

During the second altimetry decade (2002-2014) Ablain et al. (2015) have estimated that the uncertainty of the GMSL trend was lower than…..

In previous studies the uncertainty in GMSL has been estimated for long-term trends (periods of 10 years or more, starting in 1993), for inter-annual time scales (between 1 and 5 years) and annual time scales (Ablain et al., 2009; 2015).

This estimation of the uncertainty at three time-scales is a valuable first step, but it is not enough, as it does not fully meet the needs of the scientific community. In many climatic studies the GMSL uncertainty is required at different time scale and span within the 25-year altimetry record. In sea level budget studies based on the evolution of GMSL components, these estimates have been carried out at monthly time scale. In this way, the GMSL monthly changes have been interpreted in terms of changes of oceanic masses (GRACE mission).

This is also the case of studies estimating the Earth’s energy imbalance with the sea-level budget approach (Meyssignac et al., 2018). In the studies on the detection and the attribution of climate change (Slangen et al., 2017), the uncertainty in the trend estimates is needed, but over different time spans that that ones addressed by Ablain et al. (2009; 2015) and by Legeais et al. (2018). The uncertainty on different metrics is often needed. Dieng et al. (2017) and Nerem et al. (2018) have recently estimated the acceleration in the GMSL over 1993-1997, finding a small acceleration (0.08 mm/yr2) over the 25 year long altimetry record.

I suggest to eliminate this sentence, it is quite redundant.

In this paper we focus on the uncertainty in the GMSL record arising from instrumental errors in the satellite altimetry. The uncertainties of the measurements have been quantified in the GMSL record. This is an important information for the studies in the detection and attribution of the climatic changes, estimating the rise of global mean sea level as a response to the
anthropogenic activity. In the detection-attribution studies the response of the GMSL to the anthropogenic activity needs to be separated from that one to climatic variability, representing an additional source of uncertainty.

corrected

Rows 87-89

I suggest to eliminate these two sentences. They are quite redundant.

corrected

Rows 98-101

We used an error budget approach to a global scale on a 10 day basis in order to estimate the error variance-covariance matrix. We considered all the major sources of uncertainty in the altimetry measurements, including the wet tropospheric correction, the orbital solutions and the inter-calibration of satellites. We have also taken into account the time correlation between the different sources of uncertainty (section 2). The errors have been separately characterized for each altimetry mission, since they have been affected by different sources of errors (section 2).

corrected

Rows 105-106

I suggest to eliminate this sentence, it is also very redundant.

1. GMSL data series

corrected

Rows 110-117

Each group processes the 1-Hz data with geophysical corrections to correct the altimetry measurement for various aliasing, biases and drifts, caused by different atmospheric conditions, sea states, ocean tides and other causes (Ablain et al., 2009). They spatially average the data over each 10-day orbital cycle to provide GMSL estimates on a 10-day basis. The differences among the GMSL estimates from several groups arise from data editing, from difference in the geophysical corrections and from differences in the used method to spatially average the individual measurements during the orbital cycles (Masters et al., 2012; Henry et al., 2014).

corrected

Rows 117-121

Recently, the comparisons of the GMSL time series derived from satellite altimetry with independent estimates are based on the tide gauge records (Valledeau et al., 2012; Watson et al., 2015) or on the combination of the contribution to the sea level from thermal expansion, land ice melt and land-water storage (Dieng et al., 2017). They have shown that there was a drift in the GMSL record over the period 1993-1998. This drift is caused by an erroneous onboard calibration correction on TOPEX altimeter side-A (noted TOPEX-A).

corrected

2. Altimetry GMSL error budget
This section describes the different errors that affect the altimetry GMSL record. It builds on the GMSL error budget presented in Ablain et al. (2009) and extends this work taking into account the new altimeter missions (Jason-2, Jason-3) and the recent findings on altimetry error estimates.

..... and by the correlation time-scale (λ).

The biases can arise..

The drifts may occur in the GMSL record....

This drift has been corrected by using several empirical approaches (Ablain, 2017; Beckley et al., 2017; Dieng et al., 2017), that are all affected by a significant uncertainty. We estimated this uncertainty to be..... with a comparison between an independent GMSL estimate based on tide gauge records (Ablain, 2017). For the TOPEX–B record, no GMSL drift has been reported, but Ablain et al. (2012) showed significant sigma-0 instabilities in the order of 0.1 dB, which generate through the sea-state bias correction an uncertainty.....(February 1999 – April 2002). Concerning the ITRF realization Couhert et al. (2015) have shown that.......
For errors with correlation time scales lower than 2 months, we estimated the standard deviation ($\sigma$) of the error from the correction’s time series filtered with a 2-month high-pass filter. Since the standard deviation of the errors depends on the different altimeter missions, the standard deviation has been separately estimated for each altimeter mission.

In the second group of residual time correlated errors, the errors are due to the onboard microwave radiometer calibration, yielding instabilities in the wet troposphere correction and also to the orbital calculation (Couhert et al., 2015). Since these errors are correlated at a time scale longer than 5 years, they can not be estimated with the standard deviation of the correction time series, too short (25-year long) to sample the time correlation. For this group of residual-time-correlated errors we used simple models to represent the time correlation of the errors. For the wet troposphere correction, several studies (Legeais et al., 2014; Thao et al., 2014; Fernandes et al., 2015) have identified long-term differences among the computed corrections from the different microwave radiometers and from different atmospheric re-analyses (Dee et al., 2011).

3. The GMSL error variance-covariance matrix

In this section we derived the error variance-covariance matrix ($\Sigma$) of the GMSL from the error budget described in the section 2. We assumed that all the error sources shown in Table 1 are independent one to each other.

For the bias……

…not in the mean time GMSL

This is the reason because
For the drifts …..takes the shape

4. GMSL uncertainty envelope

We estimated……..

In Figure 4 we superimposed the GMSL time series (average of the GMSL time series in Figure 1) and the associated uncertainty envelop. For the TOPEX-A period we tested three different curves with three different corrections based on the removal of the Cal-1 mode (Beckley et al., 2017), on the comparison with tide gauges (Watson et al., 2015; Ablain, 2017) or based on a sea level closure budget approach (Dieng et al., 2017). The uncertainty envelop is centered on the record corrected for TOPEX-A drift with the correction based on Ablain et al. (2017). As it has been expected, all the empirically corrected GMSL records are within the uncertainty envelop.

5. Uncertainty in GMSL trend and acceleration

The variance-covariance matrix can be used to derive the uncertainty on any metric based on the GMSL time series. In this section we used the error variance-covariance matrix to estimate the uncertainty on the GMSL trend and acceleration over any period of 5 years and more within 1993-2017.

Recently, several studies (Watson et al., 2015; Dieng et al., 2017; Nerem et al., 2018; WCRP Global Sea Level Budget Group, 2018) have found a significant acceleration in the GMSL record from satellite altimetry (after correction for the TOPEX-A drift). The occurrence of an acceleration in the record should not change the estimation of the trend when calculated with a least squared approach. However, it can affect the estimation of the uncertainty on the trend. To cope with this issue, we address here at the same time both the estimation of the trend and acceleration in the GMSL record. In order to obtain this objective we used a second order polynomial as a predictor.

The most common method to estimate the GMSL trend and acceleration is the Ordinary Least Squares (OLS) estimator in its classical form (Cazenave and Llovel, 2010; Masters et al., 2012; Dieng et al., 2015; Nerem et al., 2018). This is also the most
common method to estimate trends and acceleration in other climate essential variables (Hartmann et al., 2014 and references therein).

To address this issue, we used a more general formalism to integrate the GMSL error in the trend uncertainty estimation, following Ablain et al. (2009), Ribes et al. (2016) and IPCC AR5 (Hartmann et al., 2014; see in particular Box 2.2 and Supplementary Material).

Eliminate the space

The periods for which the acceleration in sea level is significant at the 90% confidence level are shown in Fig. 8.

It is unclear which is the relationship between the acceleration of Global Mean Sea Level and the volcanic eruptions (Mount Pinatubo).

Church et al. 2005 showed that the impact of large volcanic eruptions on global ocean heat content is characterized by a rapid reduction in global ocean heat content during the year following the eruption followed by a period of recovery of a few years when global ocean heat content increases faster than before the eruption (see also Gregory et al. 2006 and Delworth et al. 2005). The sea level record starts in oct 1992 which is 1.5 years after the eruption of Mount Pinatubo (15th of June 1991). At that time the global ocean heat content was starting to recover with an increasing rate of rise (see Fasullo et al. 2016, their fig.2) leading to an acceleration in sea level. We now explain this in the text ion line 611

The period for which the trend in sea level is significant at the 90% confidence level is shown in Fig. 9. In periods when the acceleration is not significant, the second order polynomial that we used as a predictor to estimate the trend and the acceleration does not hold anymore in principle. For these periods we should turn out a first order polynomial. The use of a first order polynomial does not affect the trend estimates, but only the trend uncertainty estimates. We checked for differences in the
trend uncertainty when using either second order or first order polynomial predictors. We found that these differences are negligible (not shown). Fig. 9 indicates that for periods of 5 years and longer, the trend in GMSL is always significant at 90% CL over the whole record. At the end of the record the trend tends to increase. This is consistent with the acceleration plot in Figure 6.

6. Conclusions

……..measurement also increases and the description of the errors improves.

The uncertainty of the GMSL here computed shows the reliability of altimetry measurements in order to accurately describe the evolution of the GMSL on all time scales from 10 days to 25 years. It also shows the reliability of altimetry measurements in order to estimate the trends and the accelerations of the sea level.

In this study several assumptions have been made, that could be improved in the future. Firstly, the modeling of altimeter errors should be regularly revisited and improved to take into account a better knowledge of errors.…….. Concealing the mathematical formalism, the OLS method…….. CAPTIONS TO FIGURES

(from January 1993 to December 2017). I suggest to correct in all the captions.

Figure 1: Evolution of the GMSL series (corrected for TOPEX-A drift by using Ablain et al., 2009 TOPEX-A correction) from six different groups (AVISO/CNES, CSIRO, University of Colorado, SL_cci/ESA, NASA/GSFC, NOOA). The SL_cci/ESA covers a period from January 1993 to December 2016, while all the other products cover the full 25-year period (from January 1993 to December 2018). Seasonal (annual and semi-annual) signals have been removed and a 6-month smoothing has been applied. An averaged solution has been computed from the six groups. GMSL time series have the same average on the 1993-2015 period (common period) and the averaged solution starts at zero in 1993. The averaged solution without TOPEX-A correction has also been represented. A GIA correction of 0.3 mm/year has been subtracted to each dataset.

Figure 2: Error variance-covariance matrix of altimeter GMSL on the 25-years period (from January 1993 to December 2017).

Figure 3: Evolution in time of GMSL measurement uncertainty within a 90% confidence level (1.65 σ) on the 25-years period (from January 1993 to December 2017).
Figure 4: Evolution of the AVISO/GMSL with different TOPEX-A corrections. On the black, red and green curves, the TOPEX-A drift correction has been respectively applied based on Ablain (2017), Watson et al. (2015), Dieng et al. (2017) and Beckley et al. (2017). The uncertainty envelope, as well as the trend and acceleration uncertainties are given to a 90% confidence level (1.65 $\sigma$). Seasonal (annual and semi-annual) signals removed and 6-month smoothing have been applied. GIA correction has also been applied.

Figure 5: GMSL trend uncertainties (mm/yr) estimated for all altimeter period within a 25-years period (from January 1993 to December 2017). The confidence level is 90 % (1.65 $\sigma$). Each colored pixel respectively represents the half-size of the 90 % confidence range in the GMSL trend. The values are given in mm/yr. The vertical axis indicates the length of the period (ranging from 1 to 25 years) considered in the computation of the trend, while the horizontal axis indicates the center date of the period (for example 2000 for the 20-year period 1990-2009).

Figure 8: GMSL acceleration using the AVISO GMSL time series corrected for the TOPEX-A drift using the correction proposed by Ablain (2017): the acceleration in the shaded areas is not significant (lower than the acceleration uncertainties at the 90% confidence level). The length of the window (in years) is represented on the vertical axis and the central date of the used window is represented on the horizontal axis.

Technical corrections

Row 288  
… no observations

Row 455

SEA LEVEL BUDGET. Why capitals? Check and correct.

Row 478  
Marine Geodesy, 35 (suppl. 1), 20-41

Row 503  
Marine Geodesy, 35 (suppl. 1), 42-60.
Uncertainty in Satellite estimate of Global Mean Sea Level changes, trend and acceleration

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\textsuperscript{7}Centre National d'Études Spatiales (CNES), Toulouse, 31400, France

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Abstract. Satellite altimetry missions now provide more than 25 years of accurate, continuous and quasi-global measurements of sea level along the reference ground track of TOPEX/Poseidon. These measurements are used by different groups to build the Global Mean Sea Level (GMSL) record, an essential climate change indicator. Estimating a realistic uncertainty of the GMSL record is of crucial importance for climate studies such as estimating assessing precisely the current rate and acceleration of sea level, analyzing the closure of the sea level budget, understanding the causes of sea level rise, detecting and attributing the response of sea level to anthropogenic activity, or estimating-calculating the Earth’s energy imbalance. Previous authors have estimated the uncertainty of the GMSL trend over the period 1993-2014 by thoroughly analyzing the error budget of the satellite altimeters and have shown that it amounts to ±0.5 mm/yr (90\% confidence level). In this study, we extend our previous analysis by results providing a comprehensive description of the uncertainties in the satellite GMSL record. We analysed 25 years of satellite altimetry data and estimate-provided for the first time the error variance-covariance matrix for the GMSL record with a time resolution of 10\,-ten\, days. Three types of errors are-have been modelled (drifts, biases, noise) and combined together to derive a realistic estimate of the GMSL error variance-covariance matrix. From the error variance-covariance matrix-latter, we derived a 90\% confidence envelop of the GMSL record on a 10-day basis. Then we used a least squared approach and the error variance-covariance matrix to estimate assess the GMSL
trend and acceleration uncertainties over any 5-year time periods of 5 years and longer in between October 1992 and December 2017. Over 1993-2017, we have found a GMSL trend of 3.35±0.4 mm/yr within a 90% Confidence Level (CL) and a GMSL acceleration of 0.12 ±0.07 mm/yr² (90% CL). This is in agreement (within error bars) with previous studies. The full GMSL error variance-covariance matrix is freely available online: https://doi.org/10.17882/58344 (Ablain et al., 2018).

1 Introduction

The sea level change is a key indicator of global climate change, which integrates changes in several components of the climate-climatc system as a response to climatic variability, both anthropogenic and natural-climatc variability. Since October 1992, sea level variations have been routinely measured by twelve high-precision altimeter satellites providing more than 25 years of continuous measurements. The altimeter Global Mean Sea Level (GMSL) indicator is calculated from the accurate and stable measurements of four reference altimeter missions, namely TOPEX/Poseidon (T/P), Jason-1, Jason-2 and Jason-3. All four reference missions are flying (or have flown) over the same historical ground track on a 10-day repeat cycle. They all, have been precisely inter-calibrated (Zawadzki and Ablain, 2016) to ensure the long term stability of the sea level measurements. Six research groups (AVISO/CNES, SL_cci/ESA, University of Colorado, CSIRO, NASA/GSFC, NOAA) have processed the sea level raw data provided by satellite altimetry to estimate provide the GMSL time-series on a 10-day basis (Figure 1). The six different estimates of the GMSL record show small differences—deviations. The differences range between 1 and 2 mm at inter-annual time scales (1 to 5-year time scales) and between ±0.15 mm/yr in terms of trend over the period 1993-2017. The spread across these estimates is due to the use of different processing techniques, different alternative versions of ancillary data and different interpolation methods applied by the different several groups (Masters et al., 2012, Henry et al., 2014). This spread is smaller than the real uncertainty in the sea level trend, because all the research groups have used similar methods and corrections to process the raw data and thus several sources of systematic uncertainty are not accounted for in the spread.

In a previous study (Ablain et al., 2009) have proposed a realistic estimate of the uncertainty in the GMSL trend over 1993-2008, using an error budget approach based on the error budget. They have identified the radiometer wet tropospheric correction as one of the main sources of error. They have also identified proposed the orbital determination, the inter-calibration of altimeters and the estimate of the altimeter range, sigma-0 and significant wave height (mainly on TOPEX/Poseidon) as significant sources of error. When all the terms were accounted for, they have found that the uncertainty on the trend over 1993-2008 was ±0.6 mm/yr within a 90% confidence level. This is larger than the uncertainty of ±0.3 mm/yr over a 10-year period required by GCOS (GCOS, 2011).
framework of the ESA Sea Level Climate Change Initiative (SL_cci), significant improvements have been obtained in the estimation of the sea level from space (Ablain et al., 2015; Quartly et al., 2017; Legeais et al., 2018; Quartly et al., 2017) to get closer to the GCOS requirements. New altimeter standards including new wet troposphere corrections, new orbit solutions, new atmospheric corrections and others were selected and applied in order to improve the sea level estimation. The GMSL trend uncertainties were then updated and estimated at different temporal and spatial scales (Ablain et al., 2015; Legeais et al., 2018). During the second altimetry decade (from 2002 to 2014), Ablain et al., (2015) have estimated that the uncertainty of the GMSL trend was lower than ±0.5 mm/yr within a 90% Confidence Level (CL) for periods longer than 10 years.

In previous studies, the uncertainty in GMSL has been estimated for long-term trends (periods of 10 years or more, that starting in 1993), inter-annual time scales (at time scales between 1 and 5 years) and annual time scales (Ablain et al., 2009–2015). This estimation of the uncertainty on at three different time scales is a valuable first step, but it is not sufficient enough, as it does not fully meet the needs of the scientific community. Indeed, for many climate studies there is a need for the GMSL uncertainty is required, over different time scales and over different time spans within the 25-year altimetry record. For instance, in sea level budget studies consisting in assessing, based on the evolution of the different GMSL components, there is a need for GMSL uncertainty estimates have been carried out at monthly time scales, when we want to interpret the GMSL monthly changes have been interpreted in terms of changes of ocean mass changes (because ocean mass changes are resolved at monthly time scales since 2002 by the Gravity recovery and climate experiment (GRACE) mission). This is also the case of studies that estimating the Earth’s energy imbalance with the sea level budget approach, this is also the case (e.g. Meyssignac et al., 2018). In the studies on the detection and the attribution studies of climate change (e.g. Slangen et al., 2017), the uncertainty in the trend estimates are often needed, but over different time spans than those addressed in Ablain et al., (2009–2015) and in Legeais et al., (2018). Sometimes it is that the uncertainty on different metrics that is often needed. For example, recently Dieng et al., (2017) and Nerem et al., (2018) have recently estimated the acceleration in the GMSL over 1993-2017 and found a small acceleration of~(0.08 mm/yr²) over the 25-year-long altimetry record. There is a need for the estimation of the uncertainty in the GMSL acceleration to determine whether this acceleration is significant or not.

Note that here in this paper we focus on the uncertainty in the GMSL record that arises from the instrumental errors in the satellite altimetry instrument. This The uncertainties of the measurements can be used to have been quantified the measurement uncertainty in the GMSL record. This is an important piece of information for the studies in detection and attribution studies of the climatic changes, that seek to estimating the GMSL rise in as a response to the anthropogenic activity. But this is not sufficient. In the detection and attribution studies the...
response of the GMSL to anthropogenic activity needs to be separated from the response to the natural variability of the climate system, because the latter representing is an additional source of uncertainty. Here we do not address this problem of separating the GMSL response to different sources of variability. We strictly focus on the instrument errors and the associated uncertainty.

The objective of this paper is to estimate the error variance-covariance matrix of the GMSL (on a 10-day basis) from satellite altimetry measurements. This error variance-covariance matrix provides a comprehensive description of the uncertainties in GMSL to users. It covers all time scales that are included in the 25-year long satellite altimetry record: from 10 days (the time resolution of the GMSL time series) to multidecadal time scales. It also enables us to estimate the uncertainty in any metrics derived from GMSL measurements such as trend, acceleration or other moments of higher order in a consistent way.

To estimate the error variance-covariance matrix, we used an error budget approach at a global scale, on a 10-day basis in order to calculate the error variance-covariance matrix, in which we considered all the major sources of uncertainty in the altimetry measurements, including the wet tropospheric correction, the orbital solutions, the inter-calibration of satellites and others. We have also taken into account the temporal-time correlation between the different sources of uncertainty (section 2). The errors are have been separately characterized for each altimetry mission, since different missions are affected by different sources of uncertainty (section 2). On the basis of the error variance-covariance matrix we estimate the uncertainty in GMSL individual measurements on a 10-day basis (section 3) and the uncertainty in trend and acceleration over all periods included in the 25-year satellite altimetry record (1993-2017) (Section 4). Note that in this article all uncertainties associated to the GMSL are reported with a 90% CL unless stated otherwise.

1 GMSL data series

The six main groups that provide satellite altimetry based GMSL estimates (AVISO/CNES, SL_cci/ESA, University of Colorado, CSIRO, NASA/GSFC, NOAA) use 1-Hz altimetry measurements from T/P, Jason-1, Jason-2 and Jason-3 missions from 1993 to 2018 (1993-2015 for SL_cci/ESA). Each group processes the 1-Hz data with geophysical corrections to correct the altimetry measurements for various aliasing, biases and drifts caused, for example by different atmospheric condition, different sea states, ocean tides and others (see Ablain et al., 2009 for more details). Then they average spatially the data over each 10-day orbital cycle to provide GMSL estimates on a 10-day basis. The differences among the GMSL estimates from different groups arise from different data editing, differences in the geophysical corrections, and differences in the...
used method used to spatially average individual measurements during the orbital cycles (Masters et al., 2012; Henry et al., 2014; Masters et al., 2012).

Recently, the comparisons of the GMSL time series derived from satellite altimetry with independent estimates are based on tide gauge records (Valladeau et al., 2012; Watson et al., 2015) or on the combination of the contribution to sea level from thermal expansion, land ice melt and land water storage (Dieng et al., 2017). They have showed shown that there was a drift in the GMSL record over the period 1993-1998. This drift is caused by an erroneous onboard-board calibration correction on TOPEX altimeter side-A (noted TOPEX-A). TOPEX-A was operated from launch in October 1992 to the end of January 1999. Then TOPEX side-B altimeter (noted TOPEX-B) took over in February 1999 (Beckley et al., 2017). The impact on the GMSL changes is -1.0 mm/yr between January 1993 and July 1995, and +3.0 mm/yr between August 1995 and February 1999, with an uncertainty of ±1.7 mm/yr (within a 90%CL, (Ablain, 2017)).

Without taking into account the TOPEX-A drift correction, the differences between all GMSL time series are small. The maximum trend difference between all-time series over 1993-2017 is lower than 0.15 mm/yr, representing less than 5% of the GMSL trend. The differences observed at interannual time scales are also small (<2 mm). By correcting the drift of TOPEX-A using either of the available empirical corrections (WCRP Global Sea Level Budget Group, 2018) the differences among solutions remain the same (the difference between empirical corrections being smaller than the difference between the raw GMSL time series). Therefore, the choice of one or the other GMSL record is not decisive in this study, whose purpose is to characterize the uncertainties. Hereafter, we use the GMSL AVISO record. The corresponding altimeter standard corrections and the GMSL processing methods are described on the AVISO website (https://www.aviso.altimetry.fr/msl).

2  Altimetry GMSL error budget

This section describes the different errors that affect the altimetry GMSL record. It builds on the GMSL error budget presented in (Ablain et al. 2009) and extends this work by taking into account the new altimeter missions (Jason-2, Jason-3) and the recent findings on altimetry error estimates. Three types of errors are considered: a) biases in GMSL between successive altimetry missions which are characterized by bias uncertainties (±$\Delta$) at a given time (t); b) drifts in GMSL characterized by a trend uncertainty (±$\delta$) and c) other measurement errors which exhibit time-correlation (so called residual time correlated errors here after). The residual time correlated errors are characterized by their standard deviation ($\sigma$) and by the correlation time-scale ($\lambda$). All altimetry errors identified in this study are summarized in Erreur ! Source du renvoi introuvable. and detailed hereafter. Note that all
uncertainties reported in Table 1 are Gaussian and they are given at the 1-sigma level (i.e. we provide the standard deviation of the Gaussian, noted $\sigma$ hereafter).

The biases can arise between the GMSL record of two successive satellite missions like between T/P and Jason-1 in May 2002, Jason-1 and Jason-2 in October 2008 and between Jason-2 and Jason-3 in October 2016. These biases are estimated during dedicated 9-month inter-calibration phases when a satellite altimeter and its successor fly over the same track, 1 minute apart. During the inter-calibration phases the bias is estimated and corrected for. Different missions show different biases, but the uncertainty in the bias correction is the same for all inter-calibration phases and amounts: ±0.5 mm (Zawadzki and Ablain, 2016). The situation is different for the switch from TOPEX-A to TOPEX-B in February 1999 because it was impossible to do any inter-calibration phase between the two sides of TOPEX (as both instruments were flying on the same spacecraft). For the switch, we assume that the uncertainty in GMSL is larger and is about 2 mm (Zawadzki and Ablain, 2016).

Drifts may occur in the GMSL record because of drifts in TOPEX–A and TOPEX–B radar instruments, because of drifts in the International Terrestrial Reference Frame (ITRF) realization in which altimeter orbits are determined or because of drifts in the Glacial Isostatic Adjustment (GIA) correction applied to the GMSL record. As explained before, the TOPEX-A record shows a spurious drift due to an erroneous onboard calibration correction of the altimeter (Beckley et al., 2017). This drift has been corrected by different empirical approaches (Ablain, 2017; Beckley et al., 2017; Dieng et al., 2017), that are all affected by a significant uncertainty. With a comparison against an independent GMSL estimate based on tide gauge records (Ablain, 2017), we estimated this uncertainty to be ±0.7 mm/yr (1-$\sigma$ level) over the TOPEX-A period (1993–1998). For the TOPEX-B record, no GMSL drift has been reported, but (Ablain et al., 2012) showed significant Sigma-0 instabilities of the order of 0.1 dB, which generates through the sea-state bias correction an uncertainty of ±0.1 mm/yr (1-$\sigma$ level) in the GMSL record over the TOPEX-B period (February 1999 – April 2002). Concerning the ITRF realization, (Cohert et al., 2015) have shown that the uncertainty on the ITRF realization drift generates an uncertainty of ±0.1 mm/yr (1-$\sigma$ level) on the GMSL trend over 1993–2015. We adopt this value here for the whole period 1993–2017. For the uncertainty on the GIA correction applied to the GMSL, we use the value of 0.05 mm/yr (1-$\sigma$ level) over the altimetry period from Spada (2017). (The value is taken from the table 1 in Spada (2017). It has been confirmed recently with an ensemble of 1000 GIA runs, see Melini and Spada, 2019). Combining the uncertainty on the GMSL trend over 1993–2017 from GIA and ITRF and assuming that they are not correlated yields an uncertainty on the GMSL trend of ±0.12 mm/yr over 1993–2017 (1-$\sigma$ level). In addition to the GIA correction and the TOPEX correction, we apply an elastic correction to the GMSL record of +0.10 mm/yr to account for the elastic deformations of the ocean bottom.
in response to modern melt of land ice (Frederikse et al., 2017; Lickley et al., 2018). The uncertainty in this correction arises from the uncertainty associated to the computation of the elastic response of the solid Earth (mainly from the uncertainty associated to the procedure to solve the sea level equation, uncertainty in the choice of the Love numbers, uncertainty generated by the truncation degree of the spherical harmonics) and the uncertainty in the mass redistribution that cause the elastic deformation. Because the elastic response of the Earth is reasonably well defined (Mitrovica et al., 2011), the uncertainty in the elastic correction is largely dominated by the uncertainty in the mass redistribution (Frederikse et al., 2017). The uncertainty on the mass redistribution is about ±10% on the current ice mass loss (e.g. Blazquez et al., 2018). It yields an uncertainty of ±10% on the elastic correction (because the elastic response of the Earth is linear). This uncertainty amounts ±0.01 mm/yr which is very small. It is an order of magnitude smaller than the uncertainty considered in this study (see Table 1). So we neglect this source of uncertainty here.

The residual time correlated errors are separated into two different groups, depending on their correlation time scales. The first group gathers errors with short correlation time scales, i.e. lower than 2 months and between 2 months and one year. The second group gathers errors with long correlation time scales between 5 and 10 years. In the first group, the errors are mainly due to the geophysical corrections (e.g. ocean tides, atmospheric corrections), to the altimeter corrections (e.g. sea-state bias correction, altimeter ionospheric corrections), to the orbital calculation, and to the potential altimeter instabilities (e.g. altimeter range and sigma-0 instabilities). At time scales below one year, the variability of the corrections’ time series is dominated by errors, such that the variance of the error in each correction is estimated by the variance of the correction’s time series. For errors with correlation time scales lower than 2 months, we estimated the standard deviation ($\sigma$) of the error from the correction’s time series filtered with a 2-month high-pass filter. As the standard deviation of the errors depends on the different altimeter missions, the standard deviation has been estimated separately for each altimeter mission. We find $\sigma = 1.7$ mm over the T/P period, $\sigma = 1.5$ mm over the Jason-1 period, and $\sigma = 1.2$ mm over the Jason-2/3 period. For errors with correlation time scale between 2 months and 1 year, we used the same approach and filtered the correction time series with a pass-band filter. In this case we find $\sigma = 1.3$ mm over the T/P period, $\sigma = 1.2$ mm over the Jason-1 period, and $\sigma = 1.0$ mm over the Jason-2/3 period.

Unsurprisingly, the highest errors are obtained for T/P, and the lowest ones for Jason-2/3. This is because of: 1) larger altimeter range instabilities in T/P (Ablain et al., 2012; Beckley et al., 2017), 2) the presence of a 59-day signal error in the altimeter range of T/P (Zawadzki et al., 2018), and 3) because of the deterioration in the performance of atmospheric corrections in the early years of the altimetry era (Legeais et al., 2014). Note that Jason-1 shows also higher errors than Jason-2 and Jason-3 at time scales below 1 year (Couhert et al., 2015).

In the second group of residual time correlated errors, errors are due to the on-board microwave radiometer calibration, that yield instabilities in the wet troposphere correction, and also to the orbital calculation.
Because these errors are correlated at time scales longer than 5 years, they cannot be estimated with the standard deviation of the correction time series, the correction time series being too short (25-year long) to sample the time correlation. For this group of residual, time correlated errors, we used simple models to represent the time correlation of the errors. For the wet troposphere correction, several studies Ablain, 2017 (Fernandes et al., 2015; Legeais et al., 2014; Thao et al., 2014) have identified long-term differences among the corrections computed from the different microwave radiometers and from the different atmospheric reanalyses (e.g. ERA interim reanalyses, Dee et al., 2011). These studies report a difference in the wet tropospheric correction for GMSL in the range of ±0.2-0.3 mm/yr for periods of 5 to 10 years. Here, we adopt a conservative approach and we model the error in wet tropospheric correction with a correlated error at 5 years with a standard deviation of 1.2 mm (1 σ level). The correlation is modeled with a Gaussian attenuation based on the wavelength of the errors: $e^{-t/\lambda^2}$ with $\lambda = 5$ years. In terms of trends, this residual time correlated error generates an uncertainty of ±0.2 mm/yr over 5-year periods. For the error in the orbit calculation, comparisons of different orbit solutions showed differences of ±0.05 mm/yr on 10 year time scales due to errors in the modelling of the Earth time varying gravity field (Couhert et al., 2015). We model this error with a correlated error at 10-year time scale with a standard deviation of 0.5 mm (1-σ level). The correlation is modeled by the same Gaussian distribution as before with $\lambda = 10$ years. In terms of trends, it corresponds to an uncertainty of ±0.05 mm/yr over 10-year periods.

In the next section these different terms of the GMSL error budget are combined together to build the error variance-covariance matrix. Note that the different terms of the altimeter GMSL error budget described here are based on the current knowledge of altimetry measurement errors. As the altimetry record increases in length with new altimeter missions, the knowledge of the altimetry measurement also increases, and the description of the errors improves. This implies that the error variance-covariance matrix is expected to improve and change in the future.

### 3 The GMSL error variance-covariance matrix

In this section we derived the error variance-covariance matrix ($\Sigma$) of the GMSL from the error budget described in section 2. We assumed that all error sources presented in Table 1 are independent from one to each other. Thus the $\Sigma$ matrix is the sum of the individual variance-covariance matrix of each error source $\Sigma_i$ in the error budget (see Figure 2). Each $\Sigma_i$ matrix is calculated from a large number of random draws (> 1000) of simulated error signal using the model described in section 2 (either a bias, drift or time correlated signal) fed with a standard normal distribution.

The resulting shape of each individual $\Sigma_i$ matrix depends on the type of error (bias, drift or time correlated signal, see Figure 2). For the bias, the $\Sigma_i$ matrix takes the shape of constant square blocks each side of the time...
occurrence of the bias correction (see for example the square matrix for TOPEX-A and TOPEX-B on the low left corner of Figure 2 along the diagonal). This shape in square block means that the error in the bias correction generates an error on the GMSL which is fully correlated along time before and after the bias correction time, but which is not correlated along-time for dates that are apart of the bias correction time. This is consistent with what we expect from a bias correction error. Note that in this article (and in climate change studies in general) we are interested only in GMSL changes, trends or acceleration but not on the mean GMSL (which is the absolute reference of GMSL). Thus, we have removed from the GMSL time series the temporal mean over 1993-2017. The reference of the GMSL is thus arbitrary and assumed to be perfectly known. This is the reason why because the reference of the GMSL is not affected by the biases correction error here.

For the drifts, the $\Sigma_i$-matrix take the shape of a horse saddle. This is because an error on the GMSL drift over a given period generates errors on the GMSL time series which are correlated when they are close in time and anti-correlated when they are on opposite side of the drift period.

For residual time correlated errors, the $\Sigma_i$-matrix take the shape of a diagonal matrix with off diagonal terms of smaller amplitude. The further from the diagonal more the off-diagonal terms are, far from the diagonal the more attenuated they are. The attenuation rate is a Gaussian attenuation based on the wavelength of the time correlated errors ($e^{-\frac{1}{2}\left\langle \lambda \right\rangle^2}$), with various time-scales $\lambda$.

All individual $\Sigma_i$-matrix are summed up together to build the total error variance-covariance matrix $\Sigma$ of the altimetry-derived continuous GMSL record over 1993-2018 (see Figure 2). As expected, the dominant terms of the matrix are on the diagonal. They are largely due to the different sources of errors with correlation time scales below 1 year (first group of errors in section 2). The diagonal terms are the highest at the beginning of the altimetry period when T/P was at work. This is because of larger altimeter range instabilities in T/P, the presence of a 59-day signal error on the altimeter range of T/P and poorer performance of atmospheric corrections in the early years of the altimetry era (Legeais et al., 2014). The dominant off-diagonal terms are also found during the T/P period (in the lower left corner of the matrix, see Figure 2). The terms are induced by the TOPEX-A trend error and the large bias correction uncertainty between TOPEX-A and TOPEX-B (because of the absence of inter-calibration phase between TOPEX-A and TOPEX-B).

4 GMSL uncertainty envelope

We estimated the GMSL uncertainty envelope from the square root of the diagonal terms of $\Sigma$ (see Figure 3). As expected, the GMSL time series shows a larger uncertainty during the T/P period (5 mm to 8 mm) than during the Jason period (close to 4 mm). The bias correction uncertainty between TOPEX-A and TOPEX-B in February 1999
is also clearly visible with a 1-mm drop in the uncertainty after the switch to TOPEX side-B. Note that the uncertainty envelope has a parabolic shape and shows smaller uncertainties during the beginning of the Jason-2 period (3.5 mm around 2008) than over the Jason-3 period (4.5 mm). This is not because Jason-1 and Jason-2 errors are smaller than Jason-3's errors. Actually Jason 2 and Jason-3 errors are slightly smaller than Jason-1 errors thanks to better orbit determination. The uncertainties are smaller during the Jason-1 and Jason-2 period because this period is in the center of the record. It benefits from prior and posterior data that covariate and help in reducing the uncertainty when they are combined together. In contrast, the Jason-3 period is located at the end of the record and does not benefit from posterior data to help reduce the uncertainty.

On Figure 4 we superimposed the GMSL time series (average of the GMSL time series in Figure 1) and the associated uncertainty envelop. For the TOPEX-A period we tested three different curves with three different corrections based on the removal of the Cal-1 mode (Beckley et al., 2017), on the comparison with tide gauges (Ablain, 2017; Watson et al., 2015; Ablain, 2017), or based on a sea level closure budget approach (Dieng et al., 2017). The uncertainty envelop is centered on the corrected record for TOPEX-A drift with the correction based on (Ablain et al., 2017). As it has been expected, all the empirically corrected GMSL records are within the uncertainty envelop.

5 Uncertainty in GMSL trend and acceleration

The variance-covariance matrix can be used to derive the uncertainty on any metric based on the GMSL time series. In this section we used the error variance-covariance matrix to estimate the uncertainty on the GMSL trend and the GMSL acceleration over any period of 5 years and more within 1993-2017. Recently, several studies (Watson et al., 2015; Dieng et al., 2017; Nerem et al., 2018; Watson et al., 2015; WCRP Global Sea Level Budget Group, 2018) have found a significant acceleration in the GMSL record from satellite altimetry (after correction of the TOPEX-A drift). The presence-occurrence of an acceleration in the record should not change the estimation of the trend when estimated-calculated with a least squared approach. However, it can affect the estimation of the uncertainty on the trend. To cope with this issue, we address here at the same time both the estimation of the trend and the estimation of the acceleration in the GMSL record. In order to obtain this objective, we used a second order polynomial as a predictor. Considering the GMSL record has n observations, let X be an n × 3 predictor where the first column contains only ones (representing the constant term), the second column contains the time vector (representing the linear term) and the third column contains the square of the time vector (representing the squared term). Let y be an n × 1 vector of independent observations of the
GMSL. Let $\mathbf{e}$ be an $n \times 1$ vector of disturbances (GMSL non-linear and non-quadratic signals) and errors. Let $\mathbf{\beta}$ be the $3 \times 1$ vector of unknown parameters that we want to estimate, namely the GMSL $y$-intercept, the GMSL trend and the GMSL acceleration. Our linear regression model for the estimation of the GMSL trend and acceleration will thus be

$$y = X \mathbf{\beta} + \mathbf{e},$$

with

$$\mathbf{e} \sim N(0, \mathbf{\Sigma})$$

where $\mathbf{\Sigma}$ is the variance-covariance matrix of the observation errors (estimated in the previous section). $\mathbf{\Sigma}$ is different from the identity because of the correlated noise (see section 2).

The most common method to estimate the GMSL trend and acceleration is the Ordinary Least Squares (OLS) estimator in its classical form (Cazenave and Llovel, 2010; Masters et al., 2012; Dieng et al., 2015; Masters et al., 2012; Nerem et al., 2018). This is also the most common method to estimate trends and accelerations in other climate essential variables (Hartmann, et al., 2014 and references therein). For these reasons, we turn here to the OLS to fit the linear regression model. The estimator of $\mathbf{\beta}$ with the OLS approach, noted $\hat{\mathbf{\beta}}$ is

$$\hat{\mathbf{\beta}} = (X^\prime X)^{-1}X^\prime y.$$ 

In most cases, $\mathbf{e}$ follows a $N(0, \sigma^2 \mathbf{I})$ distribution, which implies that $\hat{\mathbf{\beta}}$ follows a Normal Law

$$\hat{\mathbf{\beta}} \sim N(\mathbf{\beta}, \sigma^2 (X^\prime X)^{-1})$$

The issue with this common framework is that the uncertainty of the trend and acceleration estimates does not take into account the correlated errors of the GMSL observations.

To address this issue, we used a more general formalism to integrate the GMSL error in the trend uncertainty estimation, following (Ablain et al., 2009; Ribes et al., 2016), and IPCC AR5 (Hartmann, et al., 2014, see in particular Box 2.2 and Supplementary Material). The OLS estimator is let unchanged (and is still unbiased), but its distribution is revised to account for $\mathbf{\Sigma}$, leading to:

$$\hat{\mathbf{\beta}} = N(\mathbf{\beta}, (X^\prime X)^{-1}(X^\prime \mathbf{\Sigma} X)(X^\prime X)^{-1})$$

Note that this estimate is known to be less accurate than the General Least Square estimate (GLS, which is the optimal estimator in the case where $\mathbf{\Sigma} \neq \mathbf{I}$) in terms of the mean square error, because its variance is larger. A generalized least square estimate would probably help in narrowing slightly the trend uncertainty but the difference would likely be small as the GMSL time series is almost linear in time. Important advantages of using OLS here are (i) OLS is consistent with previous estimators of GMSL trends as well as estimators of trends in other essential climate variable than GMSL (e.g. Hartmann, et al., 2014), and (ii) the OLS best-estimate does not depend on the estimated variance-covariance matrix $\mathbf{\Sigma}$. 

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Based on the matrix Σ defined in the previous section, and the OLS solution proposed before, we now estimate the GMSL trend (mm/yr) and acceleration (mm/yr²) uncertainties for any time span included in the period 1993-2017. Results are synthetically displayed in Figure 5 for trends and in Figure 6 for accelerations. On Figure 5, the top of the triangle indicates that the GMSL trend uncertainty over 1993-2017 is ±0.4 mm/yr (CL 90%) and that the GMSL acceleration uncertainty over the same period is ±0.07 mm/yr² (CL 90%, Figure 6). The GMSL acceleration uncertainty estimate is consistent with results of Watson et al., 2015, on the January 1993 to June 2014 time period where they find an uncertainty of ±0.058 mm yr⁻² at 1σ which corresponds to ±0.096 mm/yr² at the 90% confidence level. This is slightly larger than the Nerem et al. (2018) estimate which is ±0.025 mm/yr² at 1σ-level on the full 25-year altimetry era which corresponds to ±0.041 mm/yr² at 90% confidence level. But the Nerem et al. (2018) estimate is likely underestimated as they only consider omission errors. The GMSL acceleration uncertainties have been calculated for all periods of 10 years and more within 1993-2017 (Figure 6). As expected, uncertainties tend to increase when the period length decreases. At 10 years, the GMSL acceleration uncertainties are ranging from ±0.3 mm/yr² over the T/P period to ±0.25 mm/yr² over the Jason period. At 20 years they range between ±0.12 and ±0.08 mm/yr².
A cross-sectional analysis of the 10-year horizontal line on Figure 5 shows that the GMSL trend uncertainties over 10 years periods decreased from 1.0 mm/yr over the first decade to 0.5 mm/yr over the last one. The larger uncertainty over the first decade is mainly due to the TOPEX-A drift error, but also to the large intermission bias uncertainty between TOPEX-A and TOPEX-B and, to a lesser extent, to the improvement of GMSL accuracy with Jason-2 and Jason-3. Note that the current GCOS requirement of 0.3 mm/yr uncertainty over 10 years (GCOS, 2011) is not met at the 90% confidence level. But the recent record over the last decade based on the Jason series is close to meet the GCOS requirement with a 90% CL.

Figure 5 can also be analysed by following the sides of the triangle. The results of this analysis are plotted on Figure 7. The plain line corresponds to the left side, read from bottom left to the top of the triangle. The dashed line corresponds to the right side, read from bottom right to the top of the triangle. As expected, both curves show a reduction of the trend uncertainty as the period over which trends are computed increases from 2 to 25 years. The difference between the two lines shows the reduction of GMSL errors thanks to the improvement of the measurement in the latest altimetry missions. The lowest trend uncertainty is obtained with the last 20 years of the GMSL record: 0.35 mm/yr.

The periods for which the acceleration in sea level is significant at the 90% confidence level are shown in Figure 8. The acceleration is visible at the end of the record for periods of 10 years and longer. The GMSL acceleration is 0.12 mm/yr² with an uncertainty of 0.07 mm/yr² at 90% confidence level over the 25-year altimetry era. This proves that the acceleration observed in the GMSL evolution is statistically significant. It is worth noting that the different empirical TOPEX-A corrections yield very similar results (0.126 mm/yr² (Ablain, 2017); 0.120 mm/yr² (Dieng et al., 2017; Watson et al., 2015), 0.114 mm/yr² (Beckley et al., 2017). This acceleration at the end of the record is due to an acceleration in the contribution to sea level from Greenland and from other contributions but to a lesser extent than the acceleration recorded during the recovery from the Mount Pinatubo eruption in 1991 (Fasullo et al., 2016). Indeed, Church et al., 2005 showed that the impact of large volcanic eruptions on global ocean heat content is characterized by a rapid reduction in global ocean heat content during the year following the eruption followed by a period of recovery of a few years when global ocean heat content increases faster than before the eruption (see also Gregory et al., 2006 and Delworth et al., 2005). The sea level record starts in October 1992 which is 1.5 years after the eruption of Mount Pinatubo (15th of June 1991). At that time the global ocean heat content was starting to recover with an increasing rate of rise (see Fasullo et al., 2016, their fig.2) leading to an acceleration in sea level.
Figure 9 indicates that the period for which the trend in sea level is significant at the 90% confidence level is shown in Figure 9. In periods where the acceleration is not significant, the second order polynomial that we used as a predictor to estimate the trend and the acceleration does not hold anymore, in principle. For these periods, we should turn to a first order polynomial. The use of a first order polynomial does not affect the trend estimates, but it only affects the trend uncertainty estimates. We checked for differences in trend uncertainty when using
either second order or first order polynomial predictors. We found that these differences are negligible (not shown).

Figure 9 indicates that for periods of 5 years and longer, the trend in GMSL is always significant at 90% CL over the whole record. At the end of the record the trend tends to increase, which is consistent with the acceleration plot on Figure 6. Over the 25 years of satellite altimetry, we find a sea level rise of 3.35 ± 0.4 mm/yr (90% CL), after correcting for the TOPEX-A GMSL drift. The differences due to the different TOPEX-A corrections are negligible (<0.05 mm yr⁻¹).

6 Conclusions

In this study we have estimated the full GMSL error variance-covariance matrix over the satellite altimetry period. The matrix is available online (see section data). It provides to users a comprehensive description of the GMSL errors over the altimetry period. This matrix is based on the current knowledge of altimetry measurement errors. As the altimetry record increases in length with new altimeter missions, the knowledge of the altimetry measurement also increases, and the description of the errors improves. Consequently, the error variance-
covariance matrix is expected to change and improve in the future – hopefully with a reduction of measurement uncertainty in new products.

The uncertainty of the GMSL here computed here shows the reliability of altimetry measurements in order to accurately describe the evolution of the GMSL on all time scales from 10 days to 25 years. It also shows the reliability of altimetry measurements in order to estimate sea level trends and now-accelerations of the sea level. Along the altimetry record, we find that the uncertainty in each individual GMSL measurement decreases with time. It is smaller during the Jason era (2002-2018) than during the T/P period (1993-2002). Over the entire altimetry record, 1993-2017, we estimate the GMSL trend to $3.35 \pm 0.4$ mm/yr (90% CL, after correcting the TOPEX-A GMSL drift). We detect also a significant GMSL acceleration over the 25-year period at $0.12 \pm 0.07$ mm/yr$^2$ (90% CL).

In this study, several assumptions have been made, in this study that could be improved in the future. Firstly, the modelling of altimeter errors should be regularly revisited and improved to take into account a better knowledge of errors (e.g. stability of wet troposphere corrections) and to consider future altimeter missions (e.g. Sentinel-3 and Sentinel-6 missions). Concealing With regards to the mathematical formalism, OLS method has been applied because it is the most common approach used in the climate community to calculate trends in any climate data records. However this is not the optimal linear estimator. The use of a Generalized Least Square approach should involve some narrowing of trend or acceleration uncertainty. Another topic of concern is the consideration of the internal and forced variability of the GMSL. Here we only considered the uncertainty in the GMSL due to the satellite altimeter instrument. In a future study, it would be interesting to consider the partition of the GMSL into the forced response to anthropogenic forcing and the natural response to natural forcing and to the internal variability. Estimating the natural GMSL variability (e.g. using models) and considering it as an additional residual time correlated error, would allow to calculate the GMSL trend and acceleration representing the long-term evolution of GMSL in relationship with anthropogenic climate change.
The global mean sea level error variance-covariance matrix is available online at https://doi.org/10.17882/58344 (Ablain et al., 2018).

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References


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Global Mean Sea Level (cm)

- AVISO/CNES: trend = 3.05 mm/yr
- CSIRO: trend = 2.93 mm/yr
- University of Colorado: trend = 3.13 mm/yr
- SL_cci/ESA: trend = 2.97 mm/yr
- NASA/GSFC: trend = 3.19 mm/yr
- NOAA: trend = 3.12 mm/yr
- Averaged solution: trend = 3.10 mm/yr
- Averaged solution (no TPA correction): trend = 3.36 mm/yr
Figure 1: Evolution of GMSL time series (corrected for TOPEX-A drift using (Ablain et al., 2017) TOPEX-A correction) from six different groups (AVISO/CNES, CSIRO, University of Colorado, SL_cci/ESA, NASA/GSFC, NOAA) products. The SL_cci/ESA covers a period from January 1993 to December 2016 while all other products cover the full 25-year period (January 1993 to December 2018). Seasonal (annual and semi-annual) signals have been removed and a 6-month smoothing has been applied. An averaged solution has been computed from the six groups. GMSL time series have the same average on the 1993-2015 period (common period) and the averaged solution starts at zero in 1993. The averaged solution without TOPEX-A correction has also been represented. A GIA correction of -0.3 mm/yr has been subtracted to each data set. A correction of +0.10 mm/yr due to the deformations of the ocean bottom in response to modern melt of land ice (Frederikse et al., 2017; Lickley et al., 2018) has also been added.
Figure 2: Error variance-covariance matrix of altimeter GMSL on the 25-year period (January 1993 to December 2017).
Figure 3: Evolution in time of GMSL measurement uncertainty within a 90% confidence level (i.e., 1.65σ) on the 25-year period (January 1993 to December 2017).
Figure 4: Evolution of the AVISO GMSL with different TOPEX-A corrections. On the black, red and green curves, the TOPEX-A drift correction is has been respectively applied respectively based on Ablain, (2017), Watson et al., (2015), Dieng et al., (2017) and Beckley et al., (2017). The uncertainty envelope, as well as the trend and acceleration uncertainties are given at a 90% confidence level (i.e. 1.65σ). Seasonal (annual and semi-annual) signals removed and 6-month smoothing applied. A GIA correction of -0.3 mm/yr has been subtracted to each data set. A correction of +0.10 mm/yr due to the deformations of the ocean bottom in response to modern melt of land ice (Frederikse et al., 2017; Lickley et al., 2018) has also been added.

GIA correction has also been applied.
Figure 5: GMSL trend uncertainties (mm/yr) estimated for all altimeter periods within the 25-years period (January 1993 to December 2017). The confidence level is 90% ($\sigma = 1.65\sigma$). Each colored pixel represents respectively the half-size of the 90% confidence range in the GMSL trend. Values are given in mm/yr. The vertical axis indicates the length of the period (ranging from 1 to 25 years) considered in the computation of the trend, while the horizontal axis indicates the center date of the period (for example 2000 for the 20-year period 1990-2009).
Figure 6: GMSL acceleration uncertainties (mm/yr²) estimated for all the altimeter periods within the 25-years period (January 1993 to December 2017). The confidence level is 90 % (i.e. 1.65σ). Each colored pixel represents respectively the half-size of a 90% confidence range in the GMSL acceleration. Values are given in mm/yr². The vertical axis indicates the length of the period (ranging from 1 to 25 years) considered in the computation of the acceleration while the horizontal axis indicates the center date of the period (for example 2000 for the 20-year period 1990-2009).
Figure 7: Evolution of the GMSL trend uncertainties (within a 90% confidence level, i.e., 1.65σ) versus the altimeter period length from January 1993 to December 2017 on plain curve and from December 2017 to January 1993 on the dashed curve.
Figure 8: GMSL acceleration using the AVISO GMSL time series corrected for the TOPEX-A drift using the correction proposed by Ablain et al. (2017): the acceleration in the shaded areas is not significant (i.e., lower than the acceleration uncertainties at the 90% confidence level). The length of the window (in years) is represented on the vertical axis and the central date of the window (in years) is represented on the horizontal axis.
Figure 9: GMSL trends using the AVISO GMSL timeseries corrected for the TOPEX-A drift using the correction proposed by (Ablain, 2017). The length of the window (in years) is represented on the vertical axis and the central date of the window used (in years) is represented on the horizontal axis. A GIA correction of $-0.3 \text{ mm/yr}$ has been subtracted. A correction of $+0.10 \text{ mm/yr}$ due to the deformations of the ocean bottom in response to modern melt of land ice (Frederikse et al., 2017; Lickley et al., 2018) has also been added.
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<tr>
<td>Medium frequency errors: geophysical corrections, orbits ..</td>
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<td>σ = 1.3 mm for TOPEX period&lt;br&gt;σ = 1.2 mm for Jason-1 period.&lt;br&gt;σ = 1 mm for Jason-2/3 period.</td>
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Table 1: Altimetry GMSL error budget given at 1-sigma